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Walter Isard et José Lobo

Regional Transitions and Noise

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Walter Isard et José Lobo

Regional Transitions and Noise

Introduction

- 1 For the senior author, perhaps the most impressive aspect of Jean Paelinck's life as a scholar has been his enthusiastic undertaking of exploratory (even somewhat speculative) research and the high rate of success he has achieved in such endeavors. It is in line with his creative exploratory propensity that we undertake this paper.
- 2 For some years the senior author, as well as many others, has observed the demise of the Cold War and the collapse of the Soviet Union. He has put forth a pet theory — one emphasizing resource exhaustion under the mounting burden of increasing military expenditures. Among his fellow economists he finds other pet theories. But as most other economists, he suffers from occasional cultural blindness. Economists often fail to recognize, and typically ignore, the political, social, cultural, organizational, institutional and environmental factors strongly affecting the time evolution of economic systems. As a peace scientist, the senior author has often heard about these explanatory factors when exposed to the analysis of diverse and distinguished non-economists. He finds that within the narrow frame of each one's discipline, a non-economist's theory or hypothesis is usually reasonable and even impressive, but when examined in terms of the combined frames of social science and other disciplines, and the numerous variables noted in these frames, he remains unimpressed. From this multidisciplinary exposure, he thus easily reaches the conclusion that innumerable forces are at play, each one of which is subject to stochastic perturbations — in effect, subject to noise. Hence he and his junior co-author are motivated in this paper to reflect (or perhaps more accurately stated, speculate) on the possible relevance of stochastic perturbations for understanding the end of the Cold War and the collapse of the Soviet Union.
- 3 To do this, we must oversimplify. We shall use a model developed some 25 years ago by Isard and Liossatos (1979) to explain sudden transitions, or jumps, in urban and regional development (such as the demise of the Soviet Union). While the Isard and Liossatos model allows for the possibility of transitions among distinct economic states, the model proves inadequate in identifying the factors or forces initially responsible for the jumps. Perhaps this major shortcoming can be improved upon by introducing noise into the model.

A Simple Regional Welfare Model

- 4 We start with the model developed and discussed at length in Isard and Liossatos (1979)¹. The model postulates a *regional social welfare function* related to a single variable, x , representing population size. The function is of the form:

$$5 \quad W(x, \alpha, \beta, \gamma) = \frac{\alpha x^2}{2} + \beta x - \frac{\gamma x^4}{4}, \quad (1)$$

6 where

- 1. there are positive externalities associated with an increase in population (such as economies of scale, lower transportation costs, geographic specialization and knowledge spillovers) which are captured by the $\alpha x^2 / 2$ term;

- 2. there are deglomeration forces, or diseconomies, associated with an increase in population due to increases in traffic congestion, pollution, land rents, labor costs, crime,

etc., which we capture with the $-\gamma x^4 / 4$ term;

- 3. there is a third agglomeration force, which is strictly proportional to x —associated simply with population numbers—represented by the βx term (an absorption of stock).

7 The welfare function is depicted in Figure 1, which also shows the behavior of each of the function's additive components, for $\alpha = 3$, $\beta = 5$, and $\gamma = 1$. Note that the parameters α is set high enough relative to γ so as to limit the effect of agglomeration diseconomies until a fairly large population size is attained.

8 Clearly, higher order terms could be introduced into the welfare function presented above. Similar rationales can be presented for disaggregating agglomeration and deglomeration economies to correspond to different types of forces. For example, one type of force may correspond to localization economies, a second to urbanization economies, a third to congestion diseconomies, a fourth to pollution diseconomies, and so forth.

9 It is not necessary to discuss the obvious features of the welfare function of (1) and its components. Simply note that if the population does not increase rapidly enough, the initial stock of goods will be depleted and the initial settlement will die out, a type of event often found in the historical record.

The Deterministic Dynamics

10 We proceed to examine the equilibrium outcome from a purely deterministic standpoint. For W of equation (1) to be a maximum, we have as a first-order condition:

$$\frac{\partial W}{\partial x} = \alpha x + \beta - \gamma x^3 = 0, \quad (2)$$

11 which implies that the solutions of equation (2) are the equilibrium states for:

$$\dot{x} = \alpha x + \beta - \gamma x^3 = 0, \quad (3)$$

12 where \dot{x} stands for the time derivative of x . To study the behavior of (2) it is instructive to plot \dot{x} against x ; we do so in Figure 2. We see for $\alpha = 0$ that for each β there is a unique

x given by $x = \sqrt[3]{\beta}$. Also, for $\alpha < 0$, there is a unique x for each β . However, for

$\alpha > 0$, for each β within the interval $\left[-2\left(\frac{\alpha}{3}\right)^{\frac{3}{2}}, 2\left(\frac{\alpha}{3}\right)^{\frac{3}{2}} \right]$ there are three distinct

solutions for \dot{x} , although for values of β outside that interval there is only one.²

13 Of critical interest is the stability of any equilibrium point, inferred from the sign of the second derivative. From equation (2), we have

$$\frac{\partial^2 W}{\partial x^2} = \alpha - 3\gamma x^2. \quad (4)$$

14 Hence asymptotic stability characterizes all states for which $|x| \leq \sqrt{\alpha/3}$, since for these values $\partial^2 W / \partial x^2 \geq 0$.

15 The implications of these conditions for stability can be clearly seen by examining one of the solution curves of equation (3) in Figure 2 indicating multiple equilibrium states for some range of β . For example, we reproduce in Figure 3 that solution curve indicated by $\alpha > 0$. And in so doing, we find it convenient to interchange the axes and triple the distance representing

a unit along the β -axis. Along the lower branch $b'c'a'$, $|x| > \sqrt{\alpha/3}$; therefore

all states except a' are stable. Along the intermediate branch $a'Oa$, $|x| \leq \sqrt{\alpha/3}$; therefore all states are unstable. Hence, on the basis of this analysis alone, the system cannot stay, even under the smallest perturbations, at any equilibrium state on the intermediate branch. It then follows that for the range of β for which multiple solutions exist, and shift away from the lower branch that does not tend to return (regress) to the lower branch must end up on the upper branch, and vice versa. However, for *all small perturbations*, the property of asymptotic stability for all equilibrium points on the lower and upper branches (except for a and a') ensures that no jump from one branch to the other will occur. At a and a' small perturbations cause jumps to the other branch.

16 We obtain further analytical insight when our regional welfare function is of the specific form in equation (1), if we plot the values of W for the equilibrium states (i.e., solutions to the equation (2) of Figure 3). We then obtain Figure 4. There, only points along the two curves $a'b'$ and ab correspond to welfare maxima. The points along the curve aa' correspond to welfare minima, the points a and a' being inflection points and corresponding to the

values $\beta \pm 2(\frac{\alpha}{3})^{3/2}$. Further, we see that the welfare maximum associated with any equilibrium solution along the stretch from d' to c' is greater than the welfare maximum associated with the corresponding equilibrium solution along the stretch from a to c . Also, the welfare maximum associated with any equilibrium solution along the stretch beyond c and up to d is greater than the welfare maximum associated with the corresponding equilibrium solution along the stretch beyond c' up to a' . At c and c' , the welfare maxima are the same, being

$$\frac{\alpha}{4}$$

4. To illustrate these points, we have indicated in Figure 4 selected monetary values for the welfare function and the corresponding state values (populations) where $\alpha = 450,000$ and $x = \bar{x} - 2000$, \bar{x} being actual population.

- 17 On Figure 4, we see that an equilibrium state on the stretch from d' to c' is preferable, *from a welfare standpoint*, to its corresponding state on the stretch a to c ; and an equilibrium state on the stretch beyond c to d is preferable to a state on the stretch beyond c' to a' . At c and c' the equilibrium states are equally valued. Accordingly, if the system is moving *in macro time T* along the lower branch, it is "logical" for it to shift to the upper branch once it passes c' . But on the basis of *local stability considerations*, the system under small perturbations will continue to move along the lower branch up to point a' , and only then will it jump to the higher branch. Refer also to Figure 3. Correspondingly, on the basis of local stability considerations, if the system is on the upper branch and moving in macro time T in the direction from b to c to a , it will continue to move along the upper branch even after it passes c until it reaches a , at which point it will jump to the lower branch. Thus the local stability approach contradicts the "logical" approach based on a welfare criterion. This contradiction is due to the fact that local stability has nothing to say about global stability, that is what happens to an equilibrium state under large perturbations. Accordingly, on the basis of our welfare rationale, we can posit that all states between c' and a' on the lower branch, and all states between c and a , are stable under infinitesimal perturbations, but unstable under sufficiently large (noninfinitesimal) perturbations. We may designate these equilibrium states as *metastable*.
- 18 Observe that this situation parallels that of phase transitions in thermodynamics (from say liquid to vapor), where the phase transition should take place at points c and c' if the potential (which corresponds to our regional welfare function) is to be optimized⁵. However, both thermodynamics and our particular approach thus far developed ignore micro states, that is, the states and behavior of individual molecules and the corresponding multitude of individuals behaving units in a social system—whose behavior does not conform to the averages of the macro model. Hence we must make it explicit that equation (1) constitutes a macro-level description of a large social system containing many small individually behaving units (consumers, producers, traders, and others). Each of these units presumably maximizes some payoff and is a small system in itself.
- 19 Exactly how does the phase transition take place? Here we may quote Callen (1960), who refers to a thermodynamic system involving the macro variables volume, pressure, and temperature⁶. We may

... inquire about the manner in which the system, slowly brought to the state c' , becomes aware of the existence and attractiveness of the competing state c . We may well visualize that the system at c' , faced with the unpleasant result of an increasing Gibbs function [i.e., *decreasing welfare*] if it ventures to later its state slightly, returns from this slight exploratory excursion to the locally preferable state c' . How then is it to learn that far removed at c is an *equally attractive state*? To answer this question, we must appeal to the perspective afforded by statistical mechanics. According to statistical mechanics, the micro state of a system in equilibrium is not static, but the system perpetually undergoes rapid fluctuations. It is by means of these fluctuations that the system probes and explores to discover the state with the maximum value of the appropriate potential. To be specific, with reference to our particular system brought to the state c' , the system is not quiescent on the microscopic scale, but rather there are local fluctuations in density from point to point, the average density being that corresponding to the point c' . Of course, small fluctuations in density are most probable, and large fluctuations, such as that required to take a small portion of the system to the state c , are very rare indeed. So the system may sit for a very long time at c' , until eventually a large spontaneous fluctuation occurs, taking a small portion of the system to c . This fluctuation, unlike others, does not decay, for the portion of the system brought to c finds its new state equally satisfactory to its initial state. Such a stabilized fluctuated portion of the system then becomes a nucleus for the growth of the second phase ... In most practical instances a system brought to the state c' does not have to wait for the eventual appearance of a spontaneous fluctuation to take it to c , for artificial fluctuations are often induced by outside mechanisms. Thus a mechanical shock or tap on the system sets up longitudinal waves, with regions of condensation and rarefaction.. The region of condensation is a region of small molar volume; it is consequently a region in which the system may be brought into the state c and in which it forms a phase nucleus (Callen 1960, pp.162-163)⁷.

- 20 In parallel fashion, we can conceive the micro state of our social system in equilibrium to be not static, but perpetually undergoing rapid fluctuations. Its numerous firms, consumers, traders, organizations, governments, and planners are not average-type behaving units. Rather, there are numerous differences within the population of each of these and other classes of behaving units; they can differ greatly in the intensity and extent of their interactions with other units and their environment. Thus some of any class are always exploring —almost invariably, however, in small or smallish ways. Before the system comes to c' in macro time T , we may imagine that these explorations, even the occasional large ones, all regress to the equilibrium state. They do too, when the system is at c' , except for the very large fluctuations. But if the system sits for a long time at c' , eventually there will occur large spontaneous explorations by some units, especially under the stimulus of random shocks from the external environment. These take one or more behaving units to c . Since at c the situation is as good as at c' , those units arriving at c remain there and form the nucleus for growth of numbers at c —and beyond c if the system continues to move in the same direction.
- 21 However, this kind of interpretation is not fully satisfactory. As Callen suggests, one needs in physics to resort to statistical mechanics in order to derive a fuller explanation of transition phenomena. But this should mean that generally speaking we ought to employ probabilistic concepts in order to describe the behavior of the system when more than one attractor is present. The deterministic equation *per se* is not sufficient.
- 22 To motivate an alternative approach, consider the system at an equilibrium state on the lower branch. With respect to macro time T , let the system just cross d' from the left. As it does so, a second attractor (equilibrium state) on the upper branch becomes relevant for the system. We may then imagine that there enters, for the system, a certain amount of indeterminacy as to the equilibrium state towards which the system will regress from any random perturbation. That is, of the two and only two possible equilibrium states toward which the system will regress, there must be a positive probability for each —even though it may be extremely small for one. This indeterminacy persists until state a' is reached, provided the system has remained on the lower branch. At a' there is no indeterminacy, as there exists only one attractor, namely, d . However, it may be presumed that as the system passes from d' to c' , the probability of regressing from a random perturbation to an equilibrium state on the upper branch increases. Such a pattern may still be considered to be roughly consistent with thermodynamic (Maxwell) conventions and the Callen interpretation quoted, if the probability of occurrence of a random perturbation large enough for the system to regress to the upper branch remains very small until the system reaches the immediate neighborhood of c' . Here we may hypothesize that the probability sharply increases and almost reaches unity as the system comes close to, crosses and just passes c' . This argument, however, is only a reasonable speculation, as it is in thermodynamics. There is nothing in the equation of motion and our definitions of stability that definitely implies this interpretation or hypothesis. As in physics, in order to make more definitive statements about the behavior of the system confronting two or more alternative attractors, we can no longer rely on deterministic equations of motion to define the trajectory of the system. We must make additional assumptions that explain so-called "random" behavior in terms of well-defined probabilistic laws.

The Noisy Dynamics⁸

- 23 At this point, we should note the progress in formulating a problem that encompasses both deterministic and probabilistic behavior through the use of both a *Master Equation* and a *Fokker-Planck Equation*. First, consider the use of a Master Equation alone (Isard and Liossatos 1979, Ch.11). Assume a large system where a key property is dependent on the size of the system. Also, posit that the dynamic process governing the path of this property (its magnitude) is *Markovian*, that is where the future and past are statistically independent if the

present is known, or put otherwise, where the future is dependent on only the present and not the past. Thus, where at any time t there exists a probability distribution of states sharply peaked around the state of maximum probability, which is the mean, we become interested in an equation of motion pertaining to the mean. We become so interested because (1) we can take this equation of motion as a good approximation to the equation of motion governing the state in a purely deterministic model; and (2) such an equation of motion has the advantage of being more "realistic" since it is embedded in a stochastic process. Put otherwise, when we represent a system by deterministic equations like equation (3), we in essence ignore the effects of stochastic fluctuations. Hence our representation is not completely accurate. However, such a representation can be viewed as valid when it refers to the expected or mean values. If an equation like equation (3) is to refer to the actual instantaneous values of x , a random force term must be added to its RHS⁹.

Equivalently, however, we can associate with equation (3) a Master Equation, as developed in Isard and Liossatos (1979, pp.326-328). So to fully describe the process, we need to solve the equation of motion of the probability distribution, namely,

$$\frac{d\langle x \rangle_t}{dt} = \alpha \langle x \rangle_t + \beta - \gamma \langle x \rangle_t, \quad (5)$$

where $\langle x \rangle_t$ is the mean value of x at time t . In particular, from the Master Equation we can derive the equation of motion for the mean square deviation around any equilibrium state. If the equilibrium state is stable, the mean square deviation approaches a steady state value. However, if the equilibrium state is unstable, then the mean square deviation grows exponentially with time, a phenomenon often associated with the period of turbulence. If and when the system reaches the basin of another attractor, the mean square deviation regresses to another steady state value. In sum, the Master Equation allows us to study more closely, at least to some extent, the sequence of fluctuations not only during the normal development of a system with a stable equilibrium state, but also during the period of transition of the system from one stable state to another, or during its collapse.

Next, consider recent developments in the study of stochastic dynamics which exploit a combined Master Equation and Fokker-Planck Equation approach. One simple way to do so is to assume that the stochastic fluctuations in a system originate in the environment and directly impinge on only one element, say the α parameter of equation (3). Accordingly, at time point t :

$$\alpha_t = \langle \alpha \rangle + \sigma \xi_t, \quad (6)$$

where the bracketed term represents the mean value of α , ξ_t is Gaussian White Noise and σ , is the strength, or amplitude, of the noise.

To accommodate the two types of stochastic fluctuations and their effect on the evolution of the state variable X , we replace equation (3) with the following:

$$\frac{\partial x}{\partial t} = \langle \alpha \rangle x + \beta - x\gamma^3 + g(x)\sigma \xi_t. \quad (7)$$

When the $g(x)$ term in the above equation equals 1, or is a constant, the noise affecting x is termed *additive noise*, as when the influence of environmental fluctuations is independent of the state of the system; when $g(X)$ is not a constant and the effect of the stochastic

fluctuations does depend on the state of the system, the noise is termed *multiplicative noise* (Horsthemke and Lefever 1984). The state variable x has thus become a random variable, whose time evolution—and in particular the value it takes at a specific point of time—can only be described probabilistically by means of the probability density function $p(x, t)$.

30 As in Doering (1991), we can study the effects of different levels of noise on the behavior of equation (7) by means of time-series simulations involving extremely rapid fluctuations of an environment acting upon a steady state. To this end we discretize equation (7), a continuous equation, using the procedure outlined in Lasota and Mackey (1985). The discrete-time analogue of (7) is given by:

$$x_{t+1} = x_t + \Delta x_t \quad (8)$$

$$= x_t + \left(\langle \alpha \rangle x_t + \beta - \gamma x_t^3 + g(x_t) \sigma \omega_t \right), \quad (9)$$

31 where ω_t represents a Wiener process¹⁰. As previously stated, the stochastic fluctuations acting on the system are due to the combined effect of many undetermined factors.

32 Assuming additive noise, in particular assuming that $g(x) = 1$, and setting $x^0 = 1.04668$ at

time $t = 0$ (the steady state for one particular situation examined), with $\langle \alpha \rangle = 1$, $\beta = 0.1$, $\gamma = 1$, and $\sigma = 0.1, 0.15, 0.20$ and 0.25 , we obtain the output depicted in Figure 5. As can be expected, at low levels of noise ($\sigma = 0.1$) the state variable x experiences small fluctuations around its deterministic steady-state with an occasional jump to a negative value. When the noise amplitude is increased slightly, to 0.15, the time-series looks significantly different; not only have the deviations about the steady-states increased in magnitude, but they may suddenly jump into the range of negative x values centering around a second steady state value. In effect, the system jumps between the wells of the potential function corresponding to the welfare equation (1). Such jumps between the wells in the potential are more frequent for the time series depicted in Figure 5c ($\sigma = 0.2$). When the noise amplitude is increased even further, as in Figure 5d ($\sigma = 0.25$), the system jumps still more widely and can become explosive.

34 For multiplicative noise, where in our model $g(x) = x$, we obtain a set of graphs which, for a given steady state equilibrium point, show fluctuations of increasing magnitude with increase in the magnitude of noise and which, too, may become explosive.

35 The effects of varying noise levels on the behavior of equation (7) can also be studied *analytically* by examining how the probability density function, $p(x, t)$, changes as the noise level changes. We can do this using the *Fokker-Planck Equation*¹¹. The Ito interpretation of this equation yields, for the time-rate of change of the probability of the (x, t) associated with equations (5)-(7),

$$\frac{\partial p(x, t)}{\partial t} = \frac{\partial}{\partial x} \left[\gamma x^3 - \beta - \alpha x \right] p(x, t) + \frac{\sigma^2}{2} \frac{\partial^2}{\partial x^2} \left[g^2(x) p(x, t) \right] \quad (10)$$

36 By setting $\partial p(x, t) / \partial t$ equal to zero and then solving the Fokker-Planck Equation, we obtain the *stationary density function*, $p^s(x)$. The solution for the stationary density function has the form:

37

$$p_s(x) = Ng(x)^{-\nu} \exp\left[\frac{2}{\sigma^2} \int \frac{\alpha x + \beta - \gamma x^3}{g^2(x)} dx\right], \quad (11)$$

38 where $\nu = 1, 2$ depending on whether we assume multiplicative or additive noise, respectively, and where N is an integration constant. The behavioral effects of noise can be seen by first solving the equation for the stationary density function, and then depicting the plotting the resulting stationary density function for varying noise amplitudes. This we do in Figure 6 for additive noise.

39 Figure 6 shows the stationary density functions for $\alpha = 1, \beta = 0.1, \gamma = 1$ and varying magnitudes of σ^2 . At a small level of noise, (a) in Figure 6, the probability density has two peaks, centered around its most probable values. The two most probable values correspond to the two stable roots of the deterministic equation corresponding to equation (9) and the values of the parameters cited. These two are the roots: 1.04668 and -0.945646. The meaning of the sharpest and highest peak in the stationary density is that most of the time the system is at or near the point on which this peak is centered, with the system experiencing small fluctuations about this point. Thus for very small noise the system will still be drawn to the vicinity of its deterministic steady-state. The trough between the two peaks represents the boundary between the two wells in the potential function corresponding to equation (1). As the amplitude of the noise is increased, Figures 6b and 6c, the probability density function becomes more spread-out; the boundary between the two wells becomes less pronounced and the probability that the state variable assumes a zero value increases. For a high noise level ($\sigma^2 = 1$), as in Figure 6d, the two wells are almost equiprobable.

40 Figure 7 depicts the impact of *multiplicative noise* upon the probability densities when only the positive quadrant is considered. (There are problems of interpretation when a probability density function is considered for both positive and negative values of x' .) As is clearly evident, increase in noise leads to decrease in the maximum value of the probability density function, which value for very low magnitudes of noise lies very close to the positive deterministic steady state. As noise increases more and more, the maximum point comes to be closer and closer to the zero value for x' . However, it never really reaches it for finite values of σ^2 .

41 Of course, all the above analysis is generalizable to many variables and to situations where the transition probabilities depend explicitly on time. But even in the above single variable form it does provide a deeper insight into the stochastic nature of actual transitions. While it does not stem from the dynamic laws at the microscopic level—laws that are yet to be identified—it does provide a framework that is not too sensitive to the details of microscopic dynamics and thus permits some generality. Serious shortcomings to its application to reality reside in the many strong assumptions, such as a Markovian process, upon which it relies.

Noise and Regional Transitions: Some Speculations on the Possible Implications for the Current Situation in Europe

42 What is the significance of these simulation results and the many others that we have examined? Before answering this question, keep in mind the many behavioral and mathematical assumptions underlying the simulation runs. Moreover, we need much more time to examine the simulation results so as to obtain a firmer basis for their interpretation. Hence all we can do and all we can offer as conclusions must be viewed as highly speculative.

43 In the absence of an adequate, or generally acceptable, theory, hypothesis or speculation for explaining the collapse of the Soviet Union, it seems to us that a hypothesis stressing the cumulation of random perturbations presents a plausible and simple explanation. At a minimum it must be taken into account. However, the hypothesis does not indicate in the

innumerable perturbations taking place at any point in time when the "domain" of "regime-shifting" perturbation occurs.

44 While in figures 6 - 8 we consider perturbations that take the system's state variable from the positive to the negative domain, clearly in doing so the system must jump to, go through or jump over the $x = 0$ point. But this is so only in an abstract mathematical world. In the social-economic world, a system is unable to go beyond the point $x = 0$; at the point $x = 0$ the system must come to a total halt, which represents complete collapse. Once the $x = 0$ point is reached in the specific welfare model of equation (1), it corresponds to a locally stable position. It is clear that many societies, especially early societies, reached such a stage and disappeared. However, history records that for other societies, the transition corresponding to major decline did not reach a point of total collapse, but rather a stable equilibrium point of an earlier stage of societal development. We do not explore such a situation in our model because, as we have already noted, we choose to work within an extremely simplified framework in order to understand better the interplay of the processes involved — in particular how an unending series of stochastic perturbations takes a system to a lower stage of welfare, which in the case of equation (1) is the state $x = 0$.

45 If we were to attempt to model the dynamics of the Soviet system in the late 1980s, our set of equations would have been exceedingly complex and beyond any effective representation. Yet, from an analysis of our simple model, we can speculate that the impact of the numerous perturbations of the late 1980s did lead to a lower stage of development and welfare that had previously been reached prior to the advent of the Soviet Union.

46 Lastly, a comment about an obvious extension of the framework here presented. Having employed a stochastic differential equation to examine the behavior of a single or key region, it then follows that the same approach should be generalized so as to study the effects of noise on the behavior of a system of interconnected regions. This can be done in a fairly straightforward way. To do so here, however, would extend this paper beyond its allotted space.

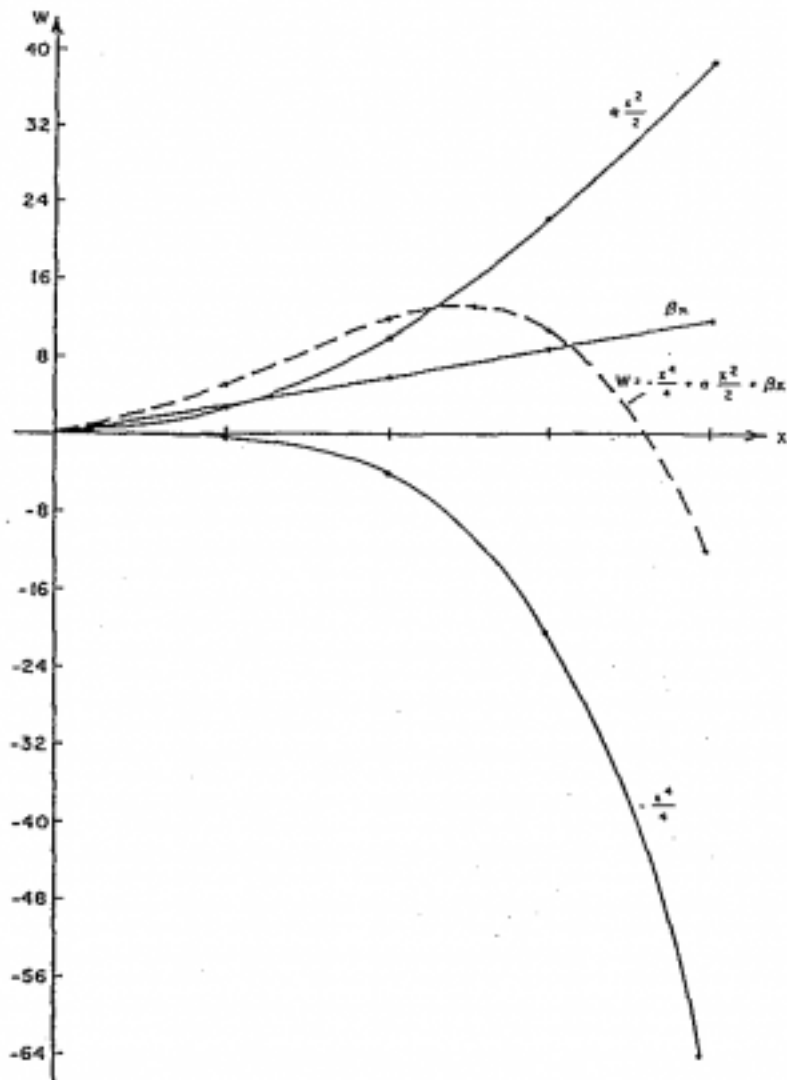


Figure 1. The terms in the Regional Welfare Function.

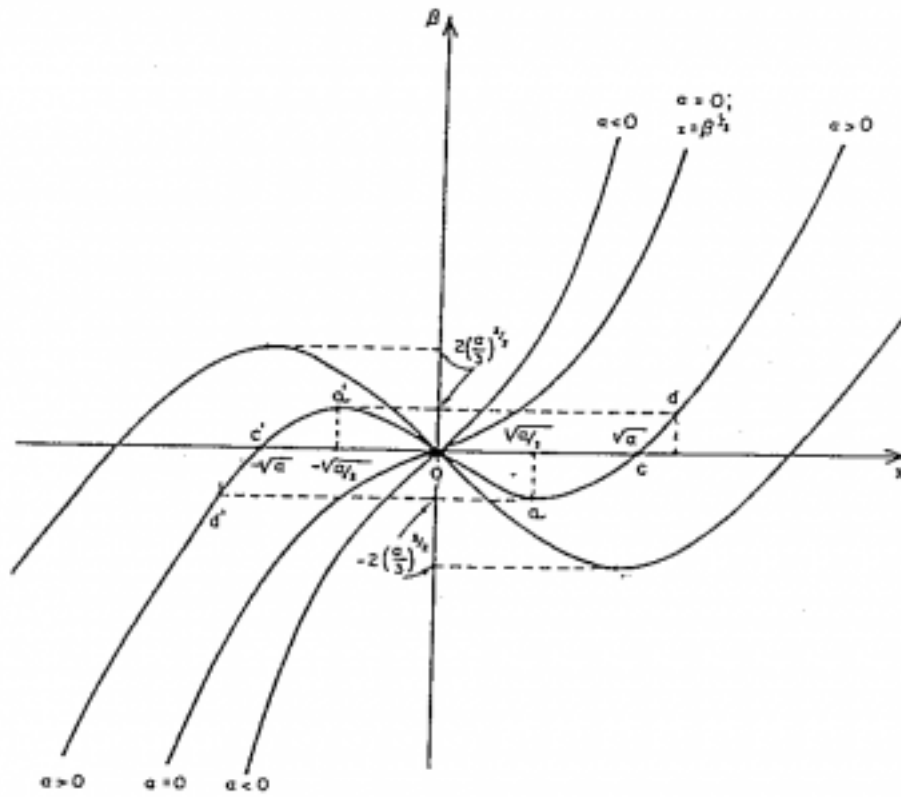


Figure 2. Variability of equilibrium States x with β for selected values of α .

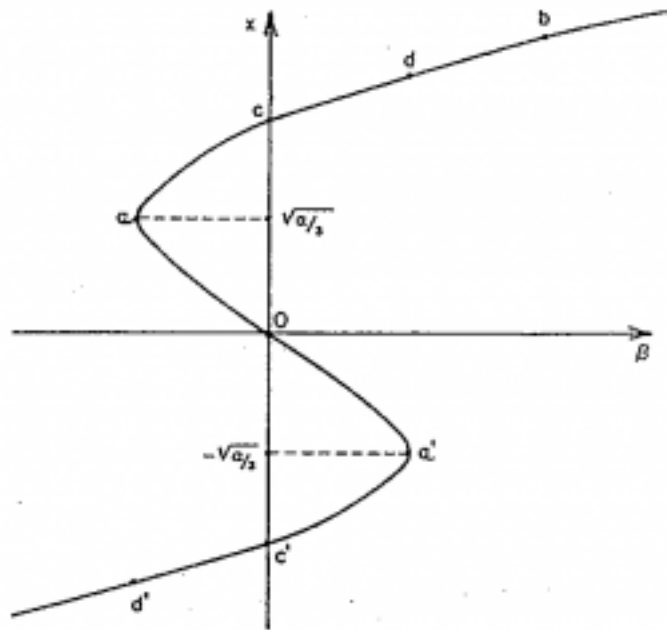


Figure 3. Variation of Equilibrium States x with β for a fixed positive value for α .

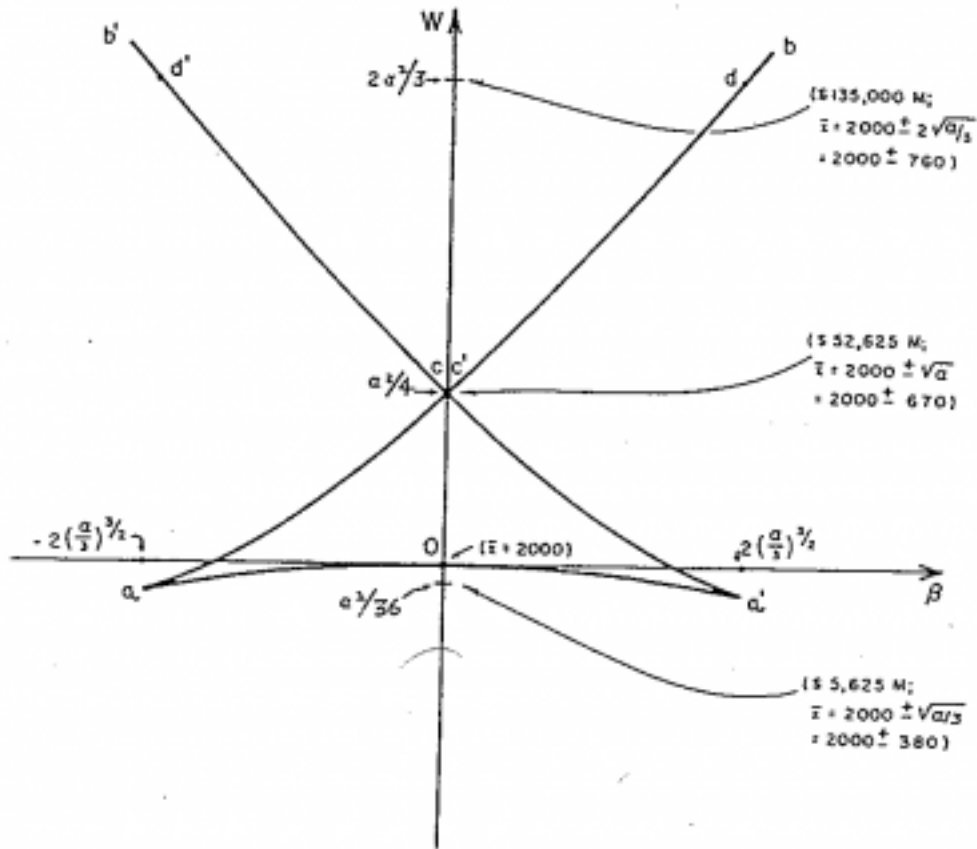


Figure 4. Variation of W with β for equilibrium states.

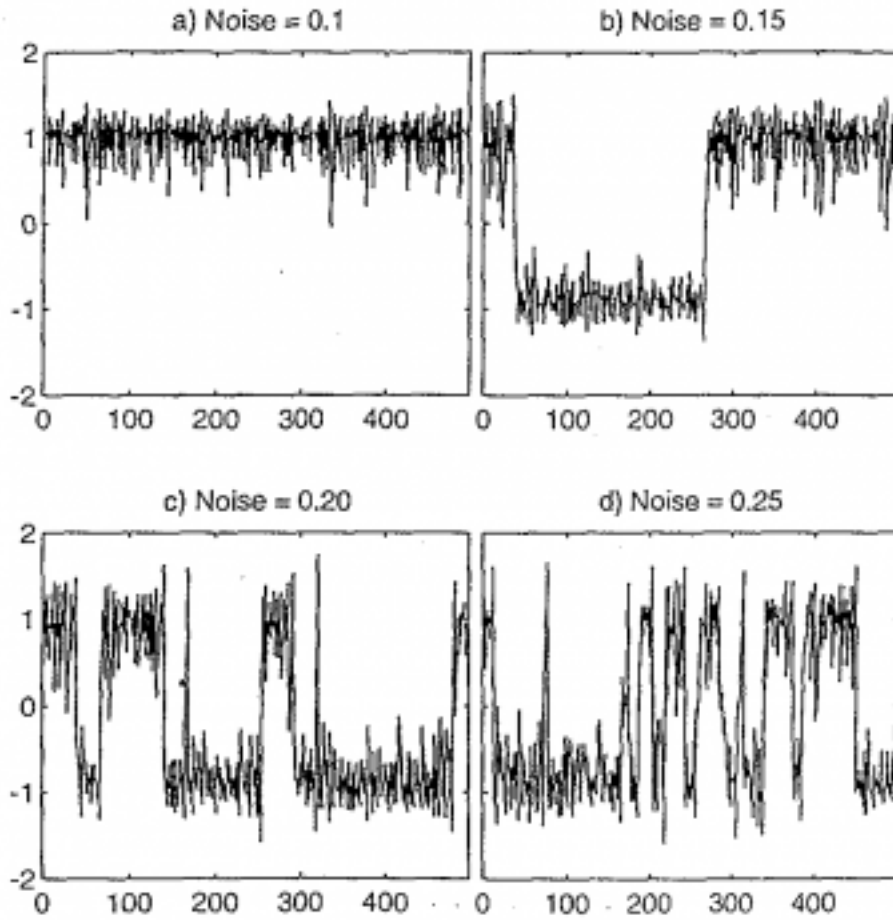


Figure 5. Time Series Simulations: Impact of Different Magnitudes of Additive Noise. (The simulations were performed for 500 units of time.)

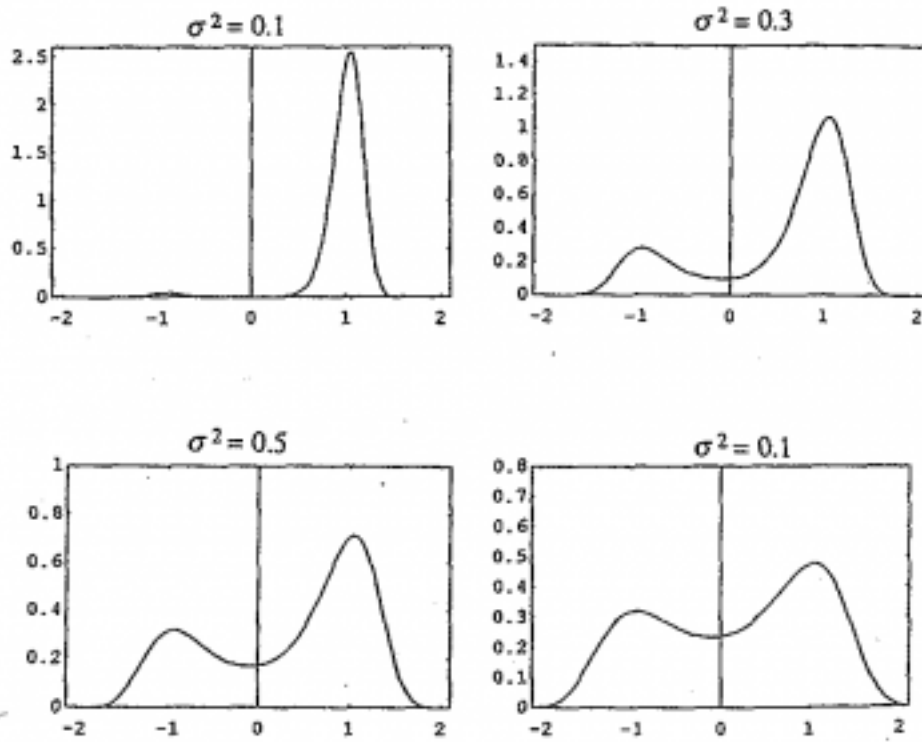


Figure 6. Some Stationary Probability Density Functions for Additive Noise.

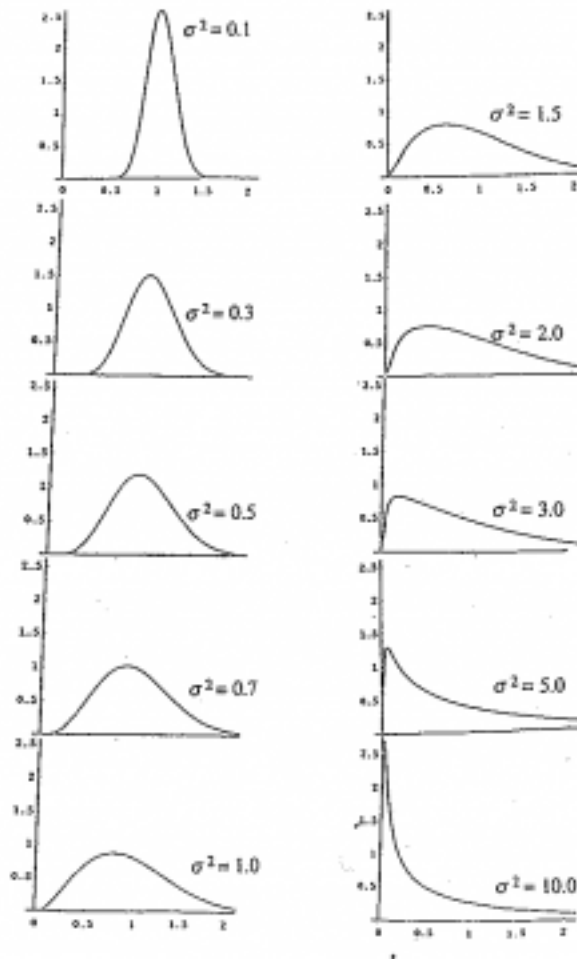


Figure 7. Some Stationary Probability Density Functions for Multiplicative Noise.

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Notes

1 Sections 2 and 3 of this paper, drawn from Isard and Liossatos (1979), provide the background model with which we will discuss the impact of noise.

2 At the boundaries of the interval, two of the three roots obviously become identical

4 In the construction of Figure 4 we have used the specific function

$$W = \frac{450,000}{2} (\bar{x} - 2000)^2 + \beta(\bar{x} - 2000) - \frac{1}{4} (\bar{x} - 2000)^4$$

where $\bar{x} = x + 2000$ and represents actual population. (Here we take the unit of x and \bar{x} as 10,000 individuals)

5 In a phase transition changes at the micro-level induce a sharp change in the macro-level properties of a substance. See Yeomans (1992)

6 A similar viewpoint can be found in Nicolis and Prigogine (1989)

7 In this quotation, an increasing Gibbs function corresponds to a decreasing welfare maximum, and the words in italics "equally attractive state" replace his "even more attractive," which we believe is more appropriate in the context of our discussion. All italics are our own.

8 The basic materials in Sections 4 and 5 were contained in the paper presented at the 1995 symposium in honor of Jean Paelinck. It appeared in the book by the senior author, *Commonalities in Art, Science and Religion* (Isard 1996), intended for a non-technical audience, highly unlikely to be read by economists and regional scientists

9 A stochastic master equation has the following form:

$$\frac{\partial}{\partial t} P(x,t) = - \sum_r \omega(x \rightarrow x+r) P(x,t) + \sum_r \omega(x \rightarrow r \rightarrow x) P(x-r,t),$$

where $P(x,t)$ is the probability that the value x is observed at time t for the state variable and

$\omega(x \rightarrow x+r)$ is the transition probability per unit of time and r is the size of the transition. For a full treatment of the Master Equation, and its derivation from the so-called Chapman-Kolmogorov equation, see van Kampen (1962)

10 A Wiener process ω^t is a stochastic process $\{\omega_t : t \geq 0\}$ such that $\omega^0 = 0$, its increments are independent and for $0 \leq s \leq t$ the random variable $\omega^t - \omega^s$ is normally distributed (Billingsley 1986). A Wiener process is typically used to simulate Gaussian White Noise.

11 For a comprehensive treatment of the Fokker-Planck equation see Risken (1989)

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Référence électronique

Walter Isard et José Lobo, « Regional Transitions and Noise », *Cybergeo : European Journal of Geography* [En ligne], Dossiers, Fragments of regional science in honour of Jean Paelinck, document 47, mis en ligne le 26 février 1998. URL : <http://cybergeo.revues.org/index346.html>

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